

Date:

Chapter: Chapter 4:4 --> Transformations w/ Matrices

Objectives: Use matrices to determine the coordinates of a translated or dilated image.

Use matrix multiplication to determine the coordinates of a reflected or rotated image.

Notes:

Video game designers often create detailed settings to add effects to games. One way they do this is to make reflections of objects on a shiny surface, such as a glass table top. To make a reflection, the game designers copy the original object and flip the copy. Matrices are frequently used to define the positions of the objects and to reposition and reorient them.

***Column Matrix** = Column of ordered pairs.

-Ex: $\begin{bmatrix} x \\ y \end{bmatrix} = (x, y)$

***Vertex/Coordinate Matrix** = Matrix made up of one or more column matrices.

-Ex: $\begin{matrix} & A & B & C \\ x & \begin{bmatrix} \\ \\ \end{bmatrix} & & \\ y & & & \begin{bmatrix} \\ \\ \end{bmatrix} \end{matrix}$

***Transformation** = Maps points of pre-image onto image.

4 Types of Transformations

- 1) Translation --> Slide (left/down (-), right/up (+))
--> Vertex Matrix + Translation Matrix
- 2) Dilation --> Shrink/Enlarge
--> Scalar[Vertex Matrix]
- 3) Rotation --> Turn (CC about 0,0)

90	180	270
$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

4) Reflection --> Flip

x-axis	y-axis	y=x
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Examples:

Ex. 1 - Determine the coordinates of the vertices of the image.

a) ABCD \rightarrow A(-5, -1) B(-2, -1) C(-1, -4) D(-3, -5)

Translation \rightarrow 3 units right, 4 units up

$$\begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \\ \text{A}' & \text{B}' & \text{C}' & \text{D}' \end{matrix}$$

$$X \begin{bmatrix} -5 & -2 & -1 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 & 0 \end{bmatrix}$$

$$Y \begin{bmatrix} -1 & -1 & -4 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 0 & -1 \end{bmatrix}$$

b) ABC \rightarrow A(-5, 4) B(-1, 5) C(-3, -1)

Translation \rightarrow 6 units left, 4 units down

Ex. 2 - Determine the coordinates of the vertices of the image.

a) ABC \rightarrow A(-5, -2) B(-3, -4) C(0, -2)

Reflection \rightarrow x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -5 & -3 & 0 \\ -2 & -4 & -2 \end{bmatrix} = \begin{bmatrix} -5 & -3 & 0 \\ 2 & 4 & 2 \end{bmatrix}$$

2x2 2x3 2x3

b) MJK \rightarrow M(-2, 3) J(4, 5) K(2, 1)

Reflection \rightarrow y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 4 & 2 \\ 3 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -2 \\ 3 & 5 & 1 \end{bmatrix}$$

c) ABCD \rightarrow A(1, 3) B(1, 7) C(3, 4) D(4, 6)

Reflection \rightarrow y=x

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 4 & 6 \\ 1 & 1 & 3 & 4 \end{bmatrix}$$

Ex. 3 - Find the coordinates of the vertices of the image.

WXYZ \rightarrow W(4, 4) X(4, 12) Y(8, 4) Z(8, 12)

Perimeter of rectangle is 1/4 of it's pre-image

$$\frac{1}{4} \begin{bmatrix} 4 & 4 & 8 & 8 \\ 4 & 12 & 4 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 3 & 1 & 3 \end{bmatrix}$$

Ex. 4 - Find the coordinates of the vertices of the image.

XYZ \rightarrow X(-5, -6) Y(-1, -3) Z(-2, -4)

Rotation \rightarrow 180 CC about 0, 0

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -5 & -1 & -2 \\ -6 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Homework:

Average (+12) --> p. 214 (#10-14 Evens, 18-34 Evens)

Advanced (+22) --> p. 214 (#10-14 Evens, 18-34 Evens,
36-40, 42-46 Evens, 49, 52)